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Design Considerations for a Digital Audio Equalizer

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DESIGN CONSIDERATIONS FOR A DIGITAL AUDIO EQUALIZER

BY

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THESIS

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ABSTRACT

The objective of this thesis is to consider a method for designing a digital audio equalizer. The primary design criteria is minimum audible frequency response error between a digital and a "reference" analog equalizer throughout the entire audio frequency range from 20 Hz to 20 Khz.

The first step is to obtain a set of analog filters that suitably represent the reference equalization. From these filters, digital filter coefficients are generated using the bilinear transformation. Then, the digital filters are combined with anti-aliasing and D/A reconstruction filters and a zero-order hold to complete the design.

Analysis of methods to minimize frequency axis warping effects on the response of the high frequency filters is presented. The problems associated with realizing a filter with low natural frequency and a very high sample rate is also studied.

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INTRODUCTION

Digital signal processing techniques are gaining widespread acceptance in the audio recording industry (Moorer 1983). The compact disc digital audio reproduction system (CD) is gaining widespread acceptance among consumers (Birchall 1985). Studio mixing desks that use all digital filtering have been in use since 1979 (Craven and Richards 1982). However, analog equalizers are still preferred over digital, apparently because digital equalizers are still more expensive to build.

There are basically two analog equalizer forms: graphic and parametric. The graphic equalizers typically consist of bandpass filters one-third to two octaves wide with fixed center frequencies and bandwidths. The system is equalized by adjusting the individual filter DC gains. Graphic equalizers generally require between ten and thirty bandpass filters. Ten fixed center frequency filters are considered to be the minimum number necessary to equalize the typical acoustic response, and the optimum number is thirty (Davis and Davis 1978). In contrast, parametric equalizers typically consist of six or less biquadratic filters with adjustable center frequencies, gains and bandwidths (Brubaker and Bullis 1981). The parametric equalizer form is more efficient in terms of the number of filters required for equalization. To take full advantage of the digital equalizer flexibility, an algorithm is required that generates the filter parameters needed to equalize an arbitrary acoustic response over the range from 20 Hz to 20 Khz.

For a digital equalizer, the most efficient form is the parametric since this form requires the fewest number of filters. The primary design consideration for the digital equalizer is to maintain the 20 Khz bandwidth of the "reference" analog equalizer. For the digital equalizer to satisfy the bandwidth specification, the lower octave filter coefficients must be realized with a high degree of precision for a parametric or graphic equalizer design.

In addition to the finite word length coefficient effects of the precision problem, there are differences between the "reference" analog and digital equalizer models due to frequency axis distortion (warping) and attenuation. The warping is due to the mapping property of the bilinear s-to-z transformation. The attenuation is due to the combination of the anti-aliasing filter, zero-order hold, and reconstruction filter.

A three-filter design is used as the "reference" analog equalizer model for comparison to the digital model. Two digital equalizer designs with 50 Khz and 100 Khz sample rates are presented. For the 50 Khz sample rate model, the effect of frequency axis distortion (warping) had to be corrected for by modifying the digital filter center frequency gain and pole Q. The 100 Khz sample rate equalizer design did not require this correction. Both of these designs begin with the development of a reference analog model, which follows.

CHAPTER I

ANALOG EQUALIZER DESIGN

Analog Model

The two forms of analog audio equalizers are the graphic and parametric. The graphic equalizers consist of bandpass filters having fixed center frequencies and bandwidths, and individually variable filter gains. The frequency response of the acoustic system is modified by adjusting the gain of each filter. To correct most typical acoustic responses, the filter networks must be placed at one-third octave intervals (Brubaker and Bullis 1981). The system block diagram of an N band graphic equalizer is shown in Figure 1. The transfer function of this system is

$$F(s) = \sum_{i=1}^{N} K_i F_i(s) \tag{1}$$

where N is the number of bandpass filter sections used.

The parametric equalizer consists of biquadratic filters. The response of an acoustic system is adjusted by varying the filter parameters. The two most common filter parameters available for modification are the center frequency gain and bandwidth. Some parametric equalizers also provide the capability to vary the filter center frequencies. The system block diagram of the parametric equalizer frequency response model that was used in this analysis is shown in Figure 2 (Bullis and Brubaker 1981). The

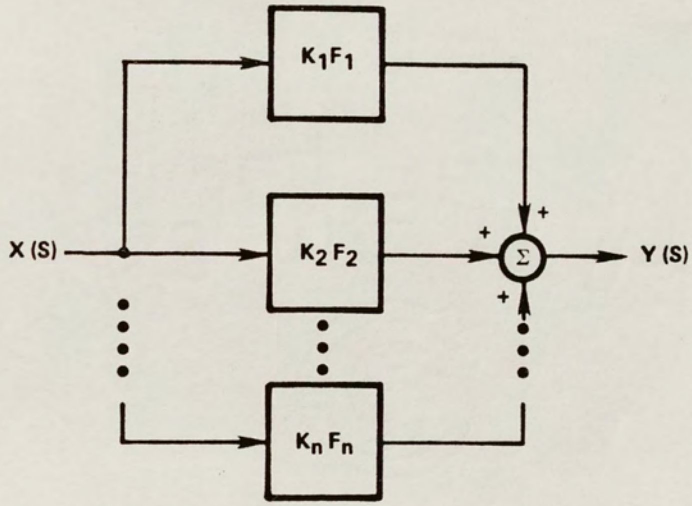

Figure 1. Graphic Equalizer Linear Model.

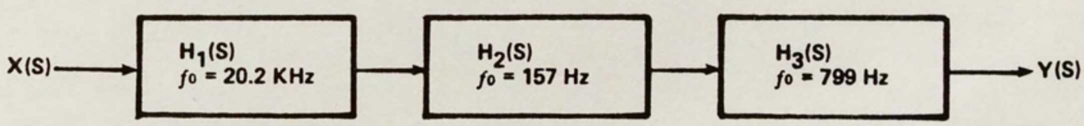

Figure 2. Parametric Equalizer Linear Model.

model consists of three biquadratic filter sections. The transfer function for this model is the product of the individual filter transfer functions which is

$$H(s) = \prod_{i=1}^{N} H_i(s) \tag{2}$$

where N is the number of parametric filter sections that are used. For the digital model that is used in the analysis, N = 3.

The transfer function for the biquadratic filter that is used in the parametric equalizer model is

$$H(s) = \frac{s^2 + s\,\dfrac{A\omega_0}{Q_p} + \omega_0{}^2}{s^2 + s\,\dfrac{\omega_0}{Q_p} + \omega_0{}^2} \tag{3}$$

The three parameters that define this transfer function are the center frequency, ω_0, the amplitude of the response at the center frequency and the pole Q. The parameter, Q_p, determines the range of frequencies affected by the filter. The frequency response magnitude of this filter may be written as

$$|H(\omega)| = \sqrt{\frac{(\omega_0{}^2 - \omega^2)^2 + \dfrac{(A\omega_0\omega)^2}{Q_p{}^2}}{(\omega_0{}^2 - \omega^2)^2 + \dfrac{(\omega\omega_0)^2}{Q_p{}^2}}} \tag{4}$$

The magnitude response of this filter at $\omega = \omega_0$ is

$$|H(\omega_0)| = A \tag{5}$$

The phase of the transfer function in radians is

$$\Phi(\omega) = \tan^{-1} \frac{A\omega\omega_0}{Q_p(\omega_0^2 - \omega^2)} - \tan^{-1} \frac{\omega\omega_0}{Q_p(\omega_0^2 - \omega^2)} \tag{6}$$

The sign of the phase term as a function of center frequency gain and frequency is given in Table 1.

TABLE 1

SECOND ORDER EQUALIZER FILTER PHASE

FREQUENCY RANGE	A > 1	A < 1
$0 < \omega < \omega_0$	+	-
$\omega > \omega_0$	-	+
$\omega = \omega_0$, 0, infinity	0	0

This form of second order filter is sometimes referred to as a bump filter because of the shape of its frequency response. Figure 3 shows frequency responses of some second order filters that have a pole Q of two and a center frequency $f_0 = 1000$ Hz. The capability to adjust each filter's center frequency allows several filters to be used where a high degree of variation exists in a given acoustic response.

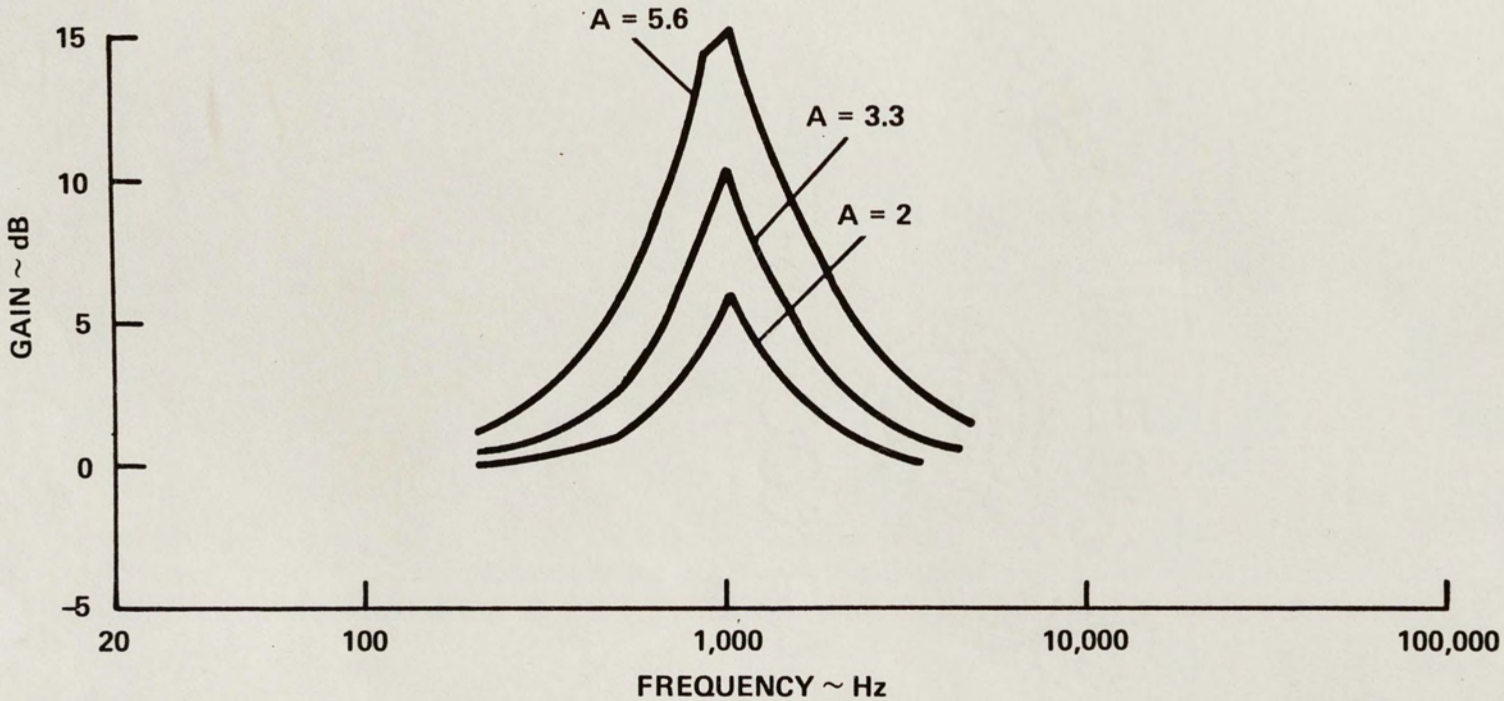

Figure 3. Second Order Equalizer Filter.

Another type of filter that is used in audio equalizers is the first order lead/lag filter. This filter is referred to as a shelving filter because of the shape of its response curve. The frequency response magnitude function of this filter is shown in Figure 4. The zero and pole frequencies for the filter shown are 7607 Hz and 5077 Hz, respectively. The lead/lag filter transfer function is

$$H(s) = \frac{s + \omega_z}{s + \omega_p} \tag{7}$$

The two parameters that control the response of this filter are ω_z, the zero frequency and the pole frequency, ω_p. The magnitude of this transfer function may be written as

$$|H(\omega)| = \sqrt{\frac{\omega^2 + \omega_z^2}{\omega^2 + \omega_p^2}} \tag{8}$$

The phase is

$$\theta(\omega) = \tan^{-1} \frac{\omega}{\omega_z} - \tan^{-1} \frac{\omega}{\omega_p} \tag{9}$$

Digital filters are obtained from the analog filters represented by equations (3) and (7) using the bilinear s-to-z transformation. The analog filter parameters are determined from acoustic response measurements using an algorithm described next.

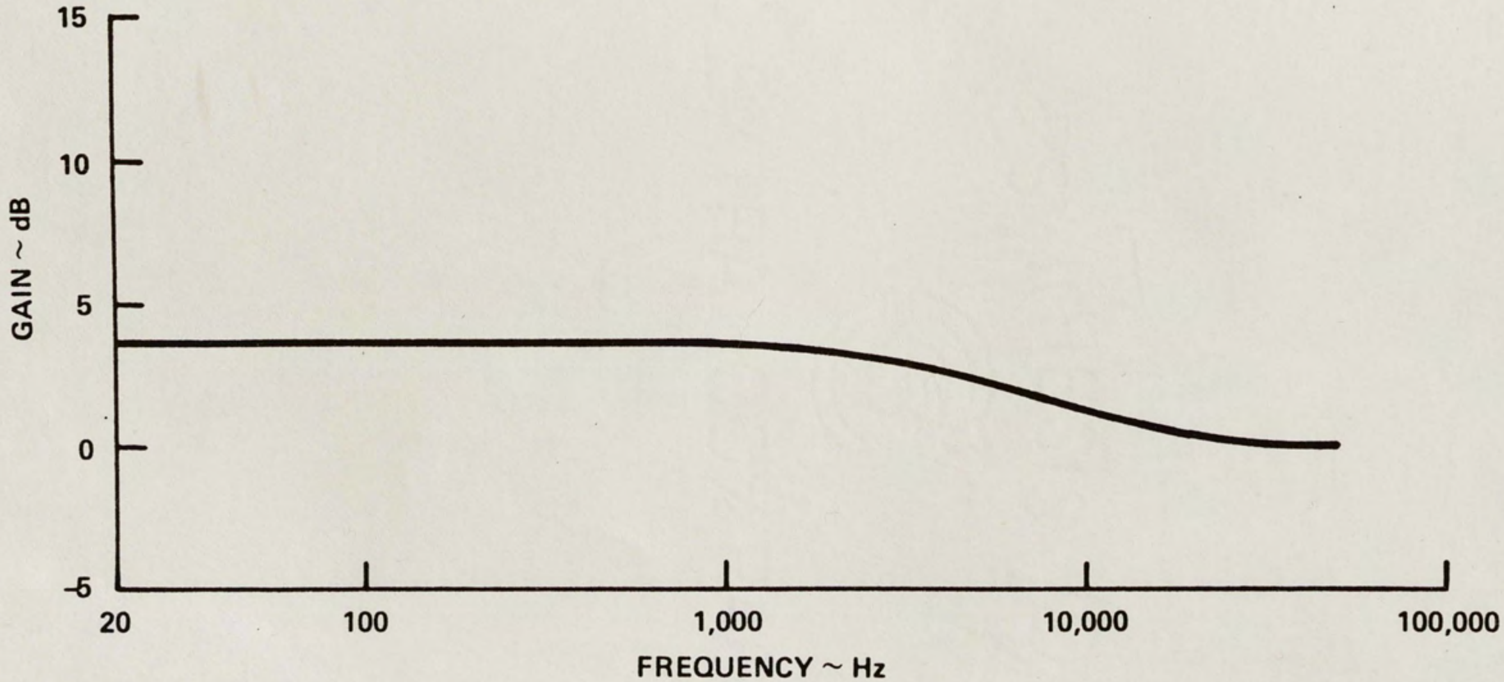

Figure 4. First Order Equalizer Filter f_z = 7607 Hz and f_p = 5077 Hz.

Filter Parameter Estimation

The reference analog equalizer filters were generated using an algorithm developed by Brubaker and Bullis (1981). The reason for choosing this algorithm is that it adequately equalizes many typical acoustic responses to within 3 to 4 dB. A general purpose automatic equalization system would include an algorithm for determining the optimum filter parameters from an arbitrary response.

An acoustic frequency response is shown in Figure 5. One characteristic common to most responses is attenuation at the high frequencies due to absorption by carpeting and furniture. Another is that acoustical responses tend to be more irregular in the 20 to 500 Hz frequency range due to standing waves (see Appendix B). The resulting filter parameters generated by the Brubaker and Bullis algorithm are given in Table 2. The filter center frequencies that equalize this acoustic response range from 50 to 20.2 Khz. The combined frequency responses of these filters and the acoustic environment is shown in Figure 6. The peak deviation has been reduced from 14 dB to 4 dB using six filters.

There are three parameters that determine the response of the biquadratic equalizer filter. The filter parameters that are estimated by the algorithm are the pole and zero Q parameters at each of the thirty measurement frequencies. The center frequency is assumed to be equal to the frequency at which the acoustic response is measured.

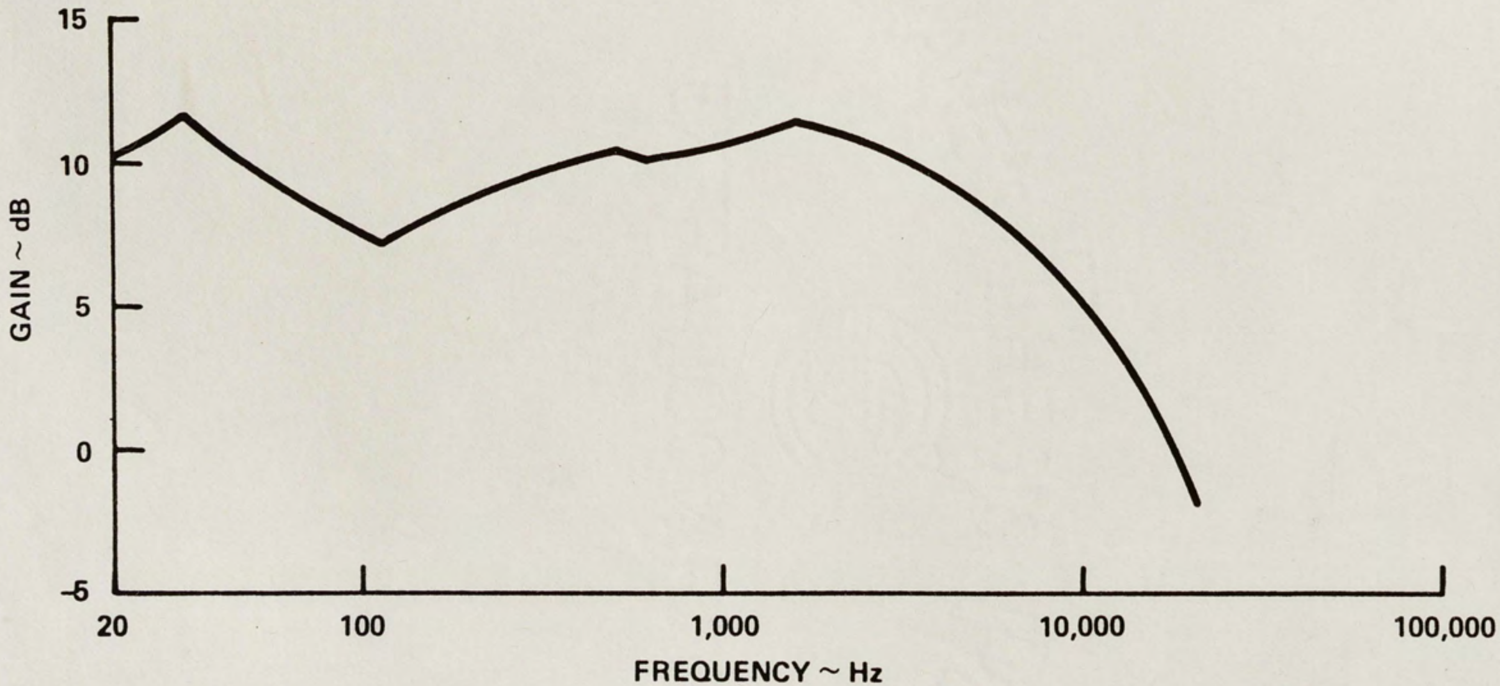

Figure 5. Acoustic Frequency Response.

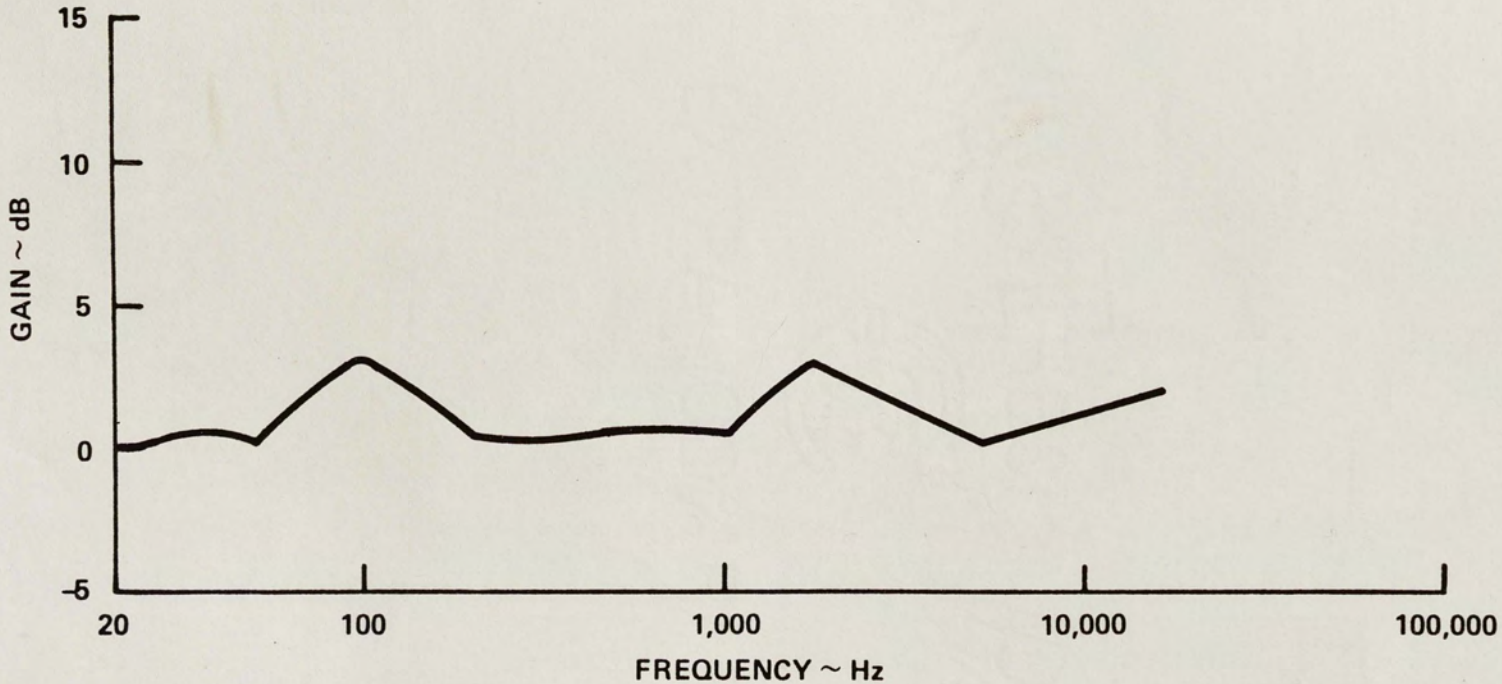

Figure 6. Combined Parametric Equalizer and Acoustic Response.

TABLE 2

ANALOG EQUALIZER FILTER PARAMETER VALUES

FILTER	ORDER	CENTER FREQUENCY (Hz)	GAIN (dB)	POLE Q
1	2	20212	14.0	1.780
2	2	157	8.8	1.160
3	2	799	6.0	1.590
4	2	503	-6.4	0.377
5	2	50	-5.2	1.400
6	1	f_z = 7607 Hz		
		f_p = 5077 Hz		

The square of the acoustic response magnitude is set equal to the biquadratic filter function.

$$R^2(\omega_n) = |H(\omega_n)|^2 \tag{10}$$

Combining equations (10) and (4) and using

$$\frac{A}{Q_p} = \frac{1}{Q_z} \tag{11}$$

we obtain

$$\overline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{Q_p} \\ \frac{1}{Q_p} \end{bmatrix} \tag{12}$$

The solution for the pole and zero Q values are found by formulating the problem with least squares. Many of the estimates may lead to an unrealizable filter, but in most cases, according to Brubaker and Bullis (1981), at least one of the thirty estimates will result in a realizable filter. When more than one estimated filter is realizable, the one that minimizes the peak deviation in the response is used.

For the first-order lead/lag filter, basically the same technique is used. Thirty first-order filters are estimated, one at each measurement frequency. An estimate of the zero frequency is made with the pole frequency set equal to each one of the measurement frequencies.

$$R^2(\omega_n) = \frac{\omega_n^{\,2} + x_1}{\omega_n^{\,2} + \omega_i^{\,2}}, \qquad \begin{array}{l} 1 \le n \le 30 \\ 1 \le i \le 30 \end{array} \tag{13}$$

$$\hat{x}_1 = \text{avg}\ (\omega_n^{\,2}(R^2(\omega_n) - 1) + \omega_i^{\,2}) \tag{14}$$

The zero frequency estimate is $\omega_z = \hat{x}_1$. The pole is $\omega_p = \omega_i$. The first-order filter estimate that minimizes the residual error in the combined response is saved. The filters generated by combining equations (13) and (15) are then compared and the one that minimizes the deviation in the response the most is selected.

The accuracy of the parameter estimation algorithm is dependent upon the accuracy with which the acoustic response is measured.

The acoustic response function is defined as $R(\omega_n)$

$$R(\omega_n) = \frac{SPL_0}{SPL_i} \tag{15}$$

where:

SPL_0 = output sound pressure level

SPL_i = reference sound pressure level

The range on the frequency index n is $1 \leq n \leq 30$.

To obtain an accurate acoustic environment response, two conditions must be met. The first is the sound pressure level (SPL) which should be measured near the expected operating level to minimize the nonlinear effects of the loudspeakers. The measurement should also be made at a sound pressure level which is at least 6 dB above the acoustic noise floor (Davis and Davis 1978). The acoustic response is measured at one-third octave intervals resulting in thirty frequency samples.

The Brubaker and Bullis algorithm is given in Figure 7 as a flowchart. The inputs to the audio equalizer parameter estimation algorithm are: the number of filters desired, desired error in the response and acoustic response data. The algorithm outputs the desired filter coefficients by computing the combination of the acoustic system and inverse filter response magnitude function. The peak deviation in this response is checked and if it is greater than desired and the maximum number of filters have not been used, another filter is estimated using the combined response function.

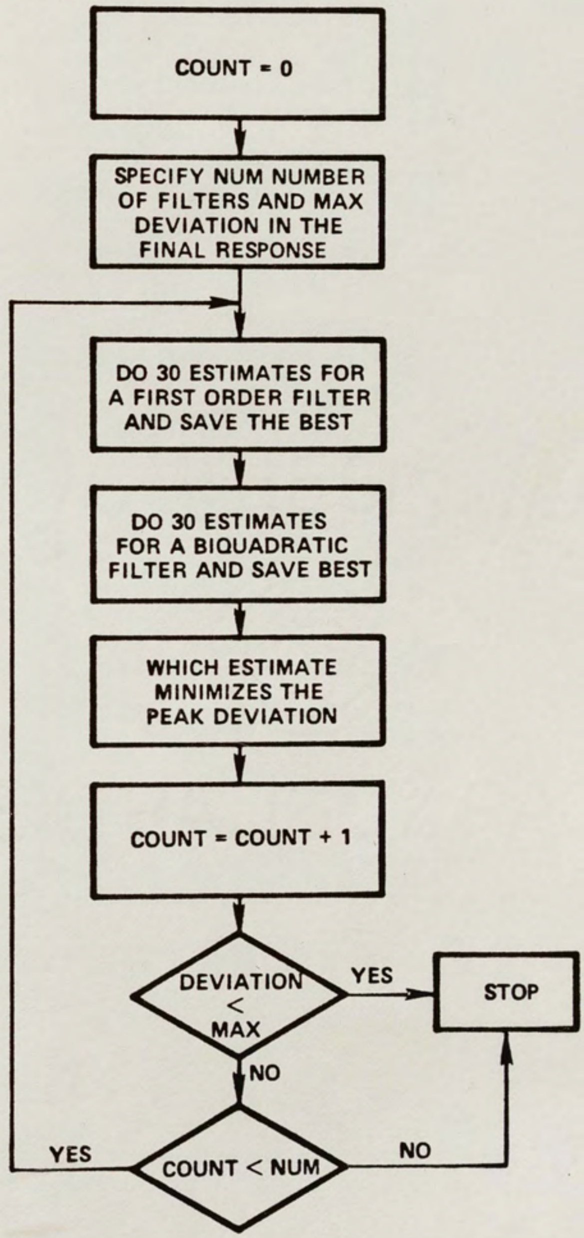

Figure 7. Parametric Filter Estimation Algorithm.

CHAPTER II

TWO DIGITAL PARAMETRIC EQUALIZER DESIGNS

Digital Model

Two digital equalizer designs are considered in this thesis. The objective in both designs is to match the response of the reference analog model presented in the previous chapter. The sample rates for the designs are 50 and 100 Khz. The primary sources of error in the digital designs are frequency axis warping and coefficient truncation effects.

The digital equalizer linear model block diagram is presented as Figure 8. This model uses ideal analog to digital (A/D) and digital to analog (D/A) converter models. The anti-aliasing filter is needed to remove any signals above one-half sample frequency. The D/A reconstruction filter is needed to help remove the sample frequency harmonics from the signal.

The well known bilinear s-to-z transformation is used to generate the digital filter coefficients because it is an excellent compromise for preserving the time and frequency domain characteristics of the analog filter. For G(s) and H(z) representing the analog and digital filter transfer functions, respectively; the bilinear s-to-z transformation is defined as

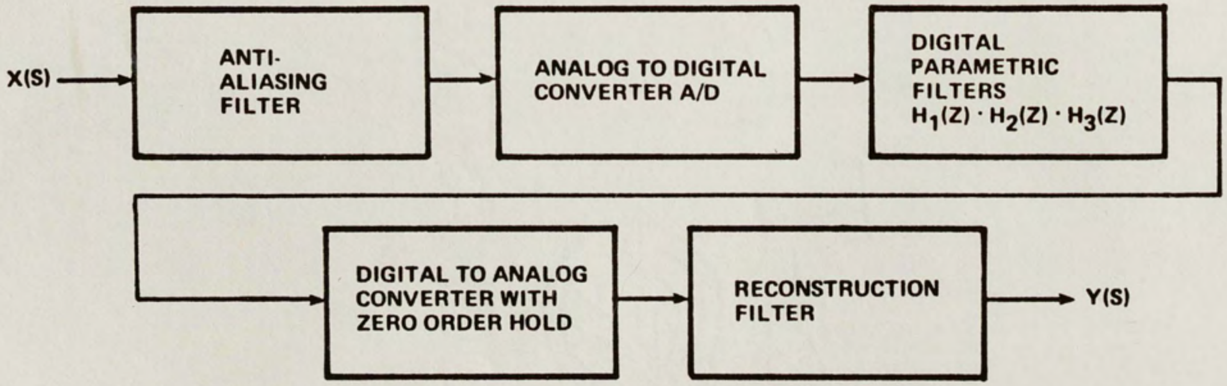

Figure 8. Digital Equalizer Linear Model.

$$H(z) = G(s) \Big|_{s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}} \tag{16}$$

where T is the digital filter sample period in seconds.

Let ω be defined as the frequency variable in the analog filter and Ω be that of the digital filter. These frequencies are related by

$$\Omega = \frac{2}{T} \tan^{-1} \frac{\omega T}{2} \tag{17}$$

The analog and digital filters will have very nearly the same amplitude response at the same frequencies if $\omega T/2$ is small because

$$\tan^{-1} \frac{\omega T}{2} \approx \frac{\omega T}{2} \tag{18}$$

Applying the bilinear transformation to a biquadratic equalizer filter

$$H(z) = \frac{s^2 + \frac{A\omega_0}{Q_p} + \omega_0{}^2}{s^2 + \frac{\omega_0 s}{Q_p} + \omega_0{}^2} \Bigg|_{s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}} \tag{19}$$

and after much algebraic manipulation, we obtain H(z)

$$H(z) = \frac{C_0 + C_1 z^{-1} + C_2 z^{-2}}{D_0 + D_1 z^{-1} + D_2 z^{-2}} \tag{20}$$

The coefficients of H(z) are related to the reference analog filter by

$$C_0 = 1 + \frac{TA\omega_0}{2Q_p} + \frac{(\omega_0 T)^2}{4} \tag{21}$$

$$D_0 = 1 + \frac{\omega_0 T}{2Q_p} + \frac{(\omega_0 T)^2}{4} \tag{22}$$

$$C_1 = \frac{(\omega_0 T)^2}{2} - 2 \tag{23}$$

$$D_1 = \frac{(\omega_0 T)^2}{2} - 2 \tag{24}$$

$$C_2 = 1 - \frac{TA\omega_0}{2Q_p} + \frac{(\omega_0 T)^2}{4} \tag{25}$$

$$D_2 = 1 - \frac{\omega_0 T}{2Q_p} + \frac{(\omega_0 T)^2}{4} \tag{26}$$

The reference analog and digital filter center frequencies are related by

$$\Omega_0 = \frac{2}{T} \tan^{-1} \frac{\omega_0 T}{2} \tag{27}$$

If $\omega_0 T/2$ is small, then the center frequencies of the reference analog and digital filters will be approximately equal and warping effects will be minimal. Rewriting $\omega_0 T/2$ as

$$\frac{\omega_0 T}{2} = \frac{2\pi f_0 T}{2} = \pi f_0/f_s \tag{28}$$

we see that small $\omega_0 T/2$ implies

$$f_s/f_0 >> \pi \tag{29}$$

The quantity f_s/f_0 is defined as the sample to center frequency ratio. As this ratio increases, the effect of warping decreases.

In order for the digital and reference analog filters' center frequencies to match, regardless of sample to center frequency ratio, a fictitious analog center frequency, ω_0, is used to derive the digital filter coefficients. This fictitious filter center frequency is

$$\omega_0' = \frac{2}{T} \tan \frac{\omega_0 T}{2} \tag{30}$$

The fictitious analog filter has the same center frequency gain and pole Q as the reference filter. The digital filter is then derived from the fictitious analog filter. Replacing ω_0 by ω_0' in equation (27), we see that the digital and reference analog center frequencies will now be equal, meaning that the derived digital filter response will match the reference analog at the desired center frequency. This technique is known as pre-warping (Antoniou 1979).

The digital filters derived in the analysis are all pre-warped to insure that they will match the reference filters at the center frequencies. If the center frequency is too near $f_s/2$, then there will be warping at frequencies close to ω_0.

High frequency response error due to warping is considered first. For this reason, the reference analog equalizer model consists of the cascade of the first three filters in Table 2 because these filters affect the high frequency response the most. Figure 9 shows the frequency responses of the 50 and 100 Khz digital equalizer filters along with the reference analog filters. The reference analog equalizer model consists of three biquadratic filter sections with center frequencies of 160, 800 and 20,200 Hz. The digital responses are equal to the reference analog at the filters centered at 160 and 800 Hz because the effect of warping is insignificant. The digital filter centered at 20.2 Khz is affected by warping significantly more at the 50 Khz rate than at the 100 Khz rate.

Considering the 20.2 Khz filter, the sample to center frequency ratio is approximately 5 for the 100 Khz sample rate and about 2.5 for the 50 Khz rate. In contrast, the ratio f_s/f_0 is about 62 for the filter centered at 800 Hz using the 50 Khz

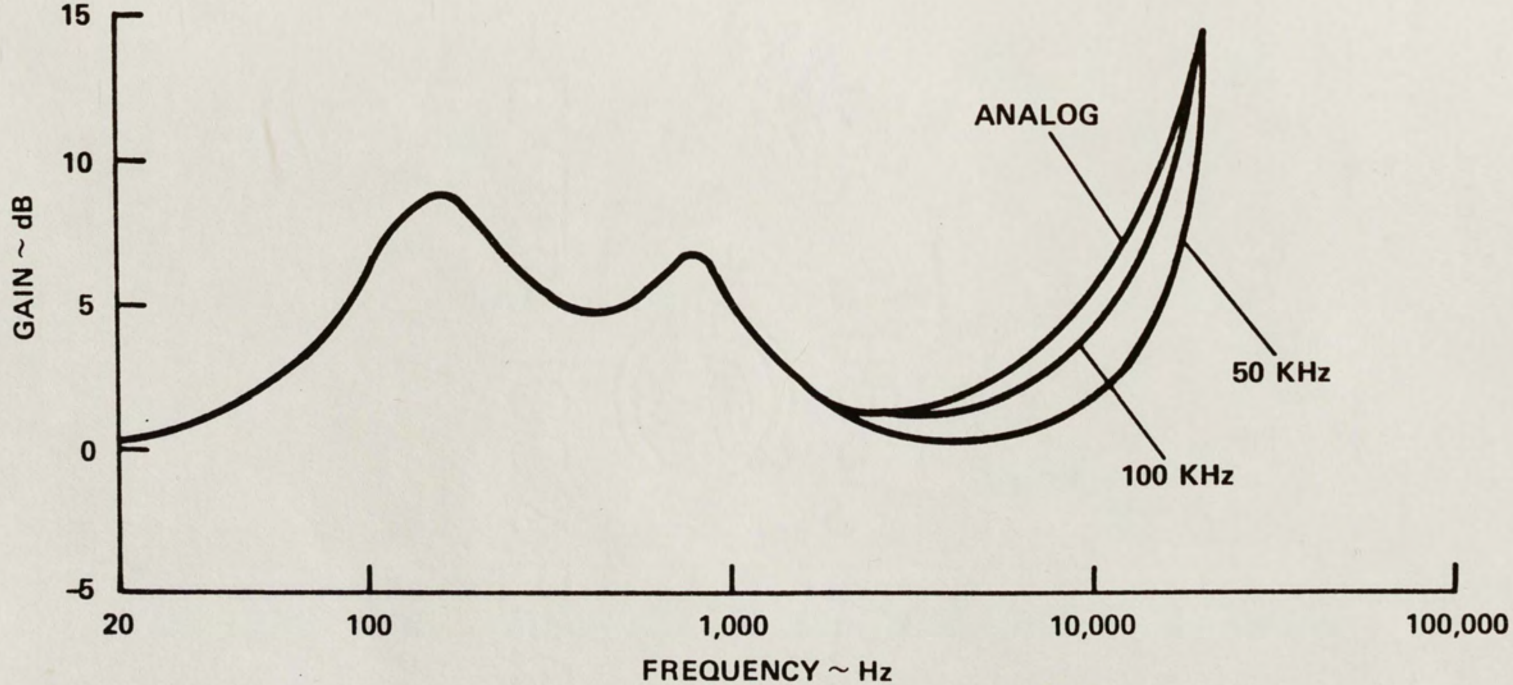

Figure 9. Digital and Analog Filter Responses.

sample rate. According to equation (29), increasing the sample to center frequency ratio decreases warping effects which explains why the 800 Hz digital and analog filters are practically identical.

Frequency axis warping is only one of the sources of digital audio equalizer frequency response error. The equalizer model also includes anti-aliasing and zero-order hold effects. It will be shown that these errors may be decreased by adjusting the parameters of the filters generated by the parametric filter estimation algorithm.

Anti-aliasing Filter Requirements

For the digital equalizer design considered here, the anti-aliasing filter is required to have minimum effect on the frequencies ranging from 20 Hz to 20 Khz. A second order low pass Butterworth filter is used in the digital equalizer design. The 20 Khz attenuation of the filter is 0.5 dB.

To realistically specify the filter requirements, listening tests could be used to determine the audible effects of aliasing. The high frequency limit for hearing should also be considered. According to listening tests, the highest audible frequency is approximately 15 Khz (Iwahara, Muraoka and Yamada 1981). For sound signals, most of the energy is below 10 Khz (see Appendix A). The energy that does exist above 10 Khz is strictly due to harmonics. Based on these facts, the bandwidth of the filter could be decreased to about 10 Khz if necessary without significant deterioration of the audio signal.

Signal Reconstruction Effect

The digital to analog converter model is an ideal zero-order hold. The transfer function of an ideal zero-order hold is

$$G_h(s) = \frac{1 - e^{-sT}}{s} \tag{31}$$

with $s = j\omega$ and $T = 2\pi/\omega_s$. The magnitude of $G_h(s)$ may be written as

$$\left| G_h(\omega) \right| = \frac{2\ \left|\sin(\omega\pi/\omega_s)\right|}{\omega} \tag{32}$$

The high frequency components resulting from the sampling process must also be removed from the equalizer output before the signal is applied to the analog system. Although the sample frequency will be much higher than any audible frequency if the harmonics are sufficiently large in amplitude, the linear range of other components in the audio system may be exceeded. At the sample frequency harmonics, the magnitude of the ideal zero-order hold model is zero, thereby eliminating the harmonics. The magnitude function is scaled such that at $\omega = 0$ it is equal to 1.0.

The frequency response magnitude in dB of the zero-order hold model used in the analysis is given in Table 3. The zero-order hold behaves like a low pass filter function with linear phase.

TABLE 3

ZERO-ORDER HOLD FREQUENCY RESPONSE

NORMALIZED FREQUENCY (f/f_s)	MAGNITUDE (dB)
0.1	-0.14
0.2	-0.6
0.3	-1.32
0.4	-2.42
0.5	-3.9
0.6	-5.9
0.7	-8.7
0.8	-12.6
0.9	-19.2
1.0	-120.0

The signal is further smoothed by a low pass filter following the zero-order hold. The function of this filter is to reconstruct the time domain signal by removing the boxcar shape from the interpolated signal, thus making the signal from the digital equalizer appear more like that of the analog equalizer. According to Tou (1959), this reconstruction filter is not needed if the signal is fairly steady which, of course, is not true for a typical audio signal. The requirements for this filter would have to be determined through listening tests.

This filter may be eliminated if the sample rate is high enough. There are CD playback systems that minimize the effects of the reconstruction filter by using oversampling. The sample rate that they employ is 186.4 Khz at the D/A converter.

For the equalizer design, the primary constraint on this filter is that it have a negligible effect on the audible frequencies. The filter used for the digital design has a gain of -1 dB at 20 Khz.

CHAPTER III

ANALOG AND DIGITAL MODEL COMPARISON

The digital and analog equalizer models are compared by the absolute value of dB gain difference in their frequency responses defined as

$$E(\omega) = \left| 20 \log |G(\omega)| - 20 \log |H(e^{j\omega T})| \right| \tag{33}$$

The frequency responses are evaluated at ω where

$$2\pi \cdot 20 \leq \omega \leq 2\pi \cdot 20K < \omega_s/2 \tag{34}$$

where $G(\omega)$ represents the analog equalizer model transfer function and $H(e^{j\omega T})$ represents the digital model function.

Response differences are considered to be significant only if they are audible. According to listening experiments, an audible difference requires a change in amplitude of about 5 dB for the average person throughout the frequency range of 100 to 11,000 Hz. The audibility of sound level changes decrease with increasing frequency. The highest audible frequency is around 15,000 Hz and the average person cannot discern the 5 dB difference in the frequency range from 11,000 to 15,000 Hz (Bucklein 1981).

The equalizer models are comprised of three biquadratic filter sections with center frequencies of 160, 800 and 20,200 Hz. The frequency response errors for the two digital models are given in Table 4. The magnitude functions for the digital and analog models are shown in Figure 10. The gain error for both digital models relative to the analog is 0 dB up to 2 Khz. The equalizers compare well at the center frequencies of the biquadratic filters located at 160 and 800 Hz because at such low frequencies the affect of tangent warping is not significant.

TABLE 4

FREQUENCY RESPONSE ERROR FOR THE DIGITAL MODELS

FREQUENCY (Hz)	100 Khz VS. ANALOG ABSOLUTE ERROR (Db)	50 Khz VS. ANALOG ABSOLUTE ERROR (dB)
20.0	0.0	0.0
157.0	0.0	0.0
800.0	0.0	0.0
5,000.0	0.4	1.6
8,000.0	1.0	3.0
10,000.0	1.2	5.0
12,000.0	1.5	6.4
14,000.0	1.8	7.6
16,000.0	2.0	8.0
18,000.0	2.0	7.0
20,200.0	2.0	4.0

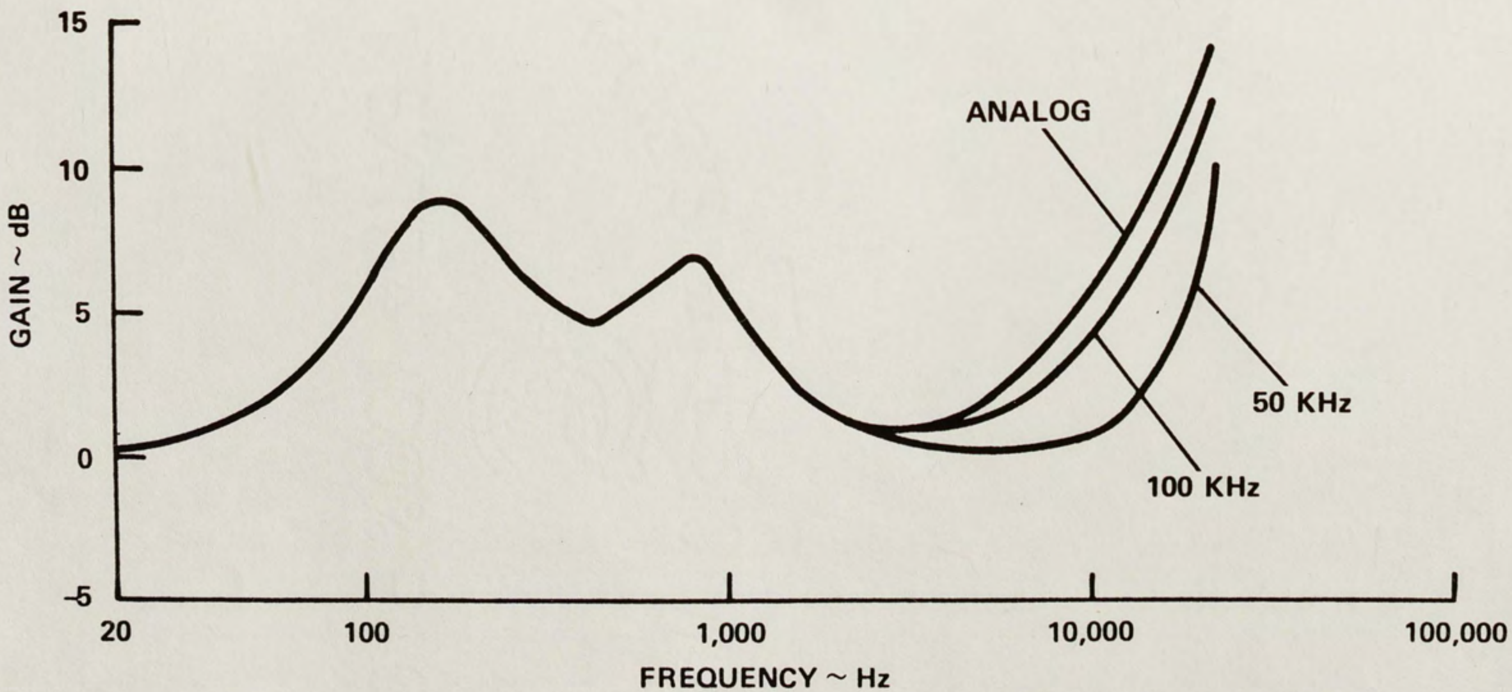

Figure 10. Analog and Digital Equalizer Model Comparison.

The response of the 50 Khz model diverges from the analog at 2 Khz. In the 100 Khz model, the error is zero up to 3,500 Hz. The peak response error of the 100 Khz model is 1.5 dB, where the peak error for the 50 Khz model is 8 dB. The advantage of the 100 Khz sample rate over the 50 Khz rate is the absence of warping in the filter at 20.2 Khz. The warping effect makes the pole Q of the 20.2 Khz filter appear to increase relative to that of the reference analog filter.

Other sources of error between the digital and the analog models are attenuation of the zero-order hold, the anti-aliasing filter and the reconstruction filter. These effects provide a net attenuation at 20 Khz of -4 dB for the 50 Khz digital model, and -2 dB for the 100 Khz model.

For the chosen "reference" analog model, the 100 Khz sample rate digital model response error should be inaudible, but using the 50 Khz sample rate, error will be audible. The effects of attenuation and warping in the 50 Khz model may be practically eliminated by increasing the center frequency gain and decreasing the pole Q of the bump filter at 20.2 Khz as given by Table 5. Applying a center frequency gain increase of 4 dB causes the error to decrease from 8 dB to 5 dB (center column). When, in addition to the gain increase, the pole Q is decreased from 1.78 to 1.0, the maximum error is reduced to 1.6 dB, which is approximately equal to the error of the unadjusted 100 Khz digital model.

TABLE 5

FREQUENCY RESPONSE ERROR OF 50 Khz
MODEL WITH 18 dB CENTER
FREQUENCY GAIN

FREQUENCY (Hz)	ABSOLUTE ERROR (dB)	
	Q_p = 1.78	Q_p = 1
1,000	0.2	0.2
5,000	1.1	0.1
10,000	3.5	0.2
12,000	4	0.7
14,000	4.2	1.0
16,000	5	1.2
18,000	4.3	0.6
20,000	1	0

In general, the frequency response error due to frequency axis warping has two solutions. One is to maintain a sample rate of about 100 Khz or more. The other solution is to first adjust the filter gain to compensate for the attenuation of zero-order hold, the anti-aliasing filter and the reconstruction filter. Then, the pole Q is adjusted to compensate for warping. By using this two-step process, it is possible to match the frequency response magnitudes of analog and digital equalizers to well within the 5 dB audible limit.

CHAPTER IV

REALIZABILITY CONSIDERATIONS

The two sources of digital equalizer response errors studied in this thesis are tangent warping and finite word length effects. Tangent warping affects the responses of filters having center frequencies that are more than about 30 percent of the sample frequency. The effect of warping is an apparent increase in the pole Q of the second order equalizer filter. Warping effects are more easily predicted and compensated for. Finite word length affects the coefficients and the arithmetic operations. These types of errors do not have easily predictable results because they depend on the value of the filter coefficients.

Many filter architectures have been proposed to minimize the effects of the coefficient quantization problem. The direct form realization of a digital filter is the most sensitive to coefficient quantization (McCallig, Kurth and Steel 1979). Coefficient quantization effects are much more significant for biquadratic equalizer filters with center frequencies that are much less than the sample frequency. This is consistent with the fact that low pass digital filters having cut-off frequencies much less than the sample frequency are also more sensitive to coefficient errors.

To gain insight into the problems caused by coefficient quantization, digital filter coefficients are computed with double precision accuracy and then quantized to the precision of 16 binary digits. Full precision is assumed for the arithmetic filter operations throughout this coefficient sensitivity analysis.

Coefficient Quantization Effects

The digital second order filter transfer function is

$$H(z) = \frac{C_0 + C_1 z^{-1} + C_2 z^{-2}}{D_0 + D_1 z^{-1} + D_2 z^{-2}} \tag{35}$$

The difference equation resulting from applying the inverse Z transform to H(z) is

$$D_0 y(n) = \sum_{k=0}^{2} C_k x(n-k) - \sum_{k=1}^{2} D_k y(n-k) \tag{36}$$

Digital filters are normally implemented with $D_0 = 1$. The coefficients of the difference equation are normalized by D_0.

The binary number is represented with 1 sign bit, I integer bits and f fractional bits symbolically as

$$SI \cdot f$$

The scale factor is determined from

$$SF \cdot U = 2^{N-1} - 1 \tag{37}$$

where:

SF = the scale factor

U = the smallest power of 2 that is greater than the largest filter coefficient

N = 1 + I + f (where N is the total number of bits including sign)

For a biquadratic filter, the maximum coefficient magnitude is less than 2, therefore U = 2

$$SF = \frac{(2^{N-1} - 1)}{U} \cong 2^{N-1}/U$$

Normally, the maximum value U is an integer power of 2.0. For this case, the scale factor is

$$SF = 2^{N-2} = 2^{14} \tag{38}$$

The quantization level is 1/SF. The biquadratic filter coefficients are quantized to an accuracy of $1/SF = 2^{-14}$ = 0.000061035 using 16 bit precision. The algorithm used for quantizing the coefficients is

$$\begin{aligned} C_r &= C_t + 2^{-f}, && \text{if } C - C_t > 2^{-(f+1)} \\ C_r &= C_t, && \text{otherwise} \end{aligned} \tag{39}$$

where:

C = floating point value of coefficient

C_t = truncated value of coefficient (negative values rounded down)

C_r = quantized coefficient 16 bit precision

f = number of fractional bits

The frequency response for the combination of the three lowest frequency biquadratic filters of Table 2 with quantized coefficients is shown in Figure 11. The center frequencies are 50, 157 and 800 Hz. The response at the sample rate of 50 Khz error is 5.5 dB at the 50 Hz center frequency filter.

The response curves for a 20 Hz biquadratic equalizer filter at 35 and 50 Khz sample rates are given in Figure 12. The 35 Khz rate filter has a center frequency at 35 Hz. The 50 Khz rate filter response is practically flat. In contrast to these results, using full double precision in BASIC (13 digits), a 20 Hz filter can be represented at a sample rate of 50 Khz with less than 1 dB error from 20 Hz to 20 Khz.

Using these frequency responses, an approximate range on the sample to center frequency ratio to achieve a 2 dB frequency response error is

$$5 < f_s/f_0 < 500 \tag{40}$$

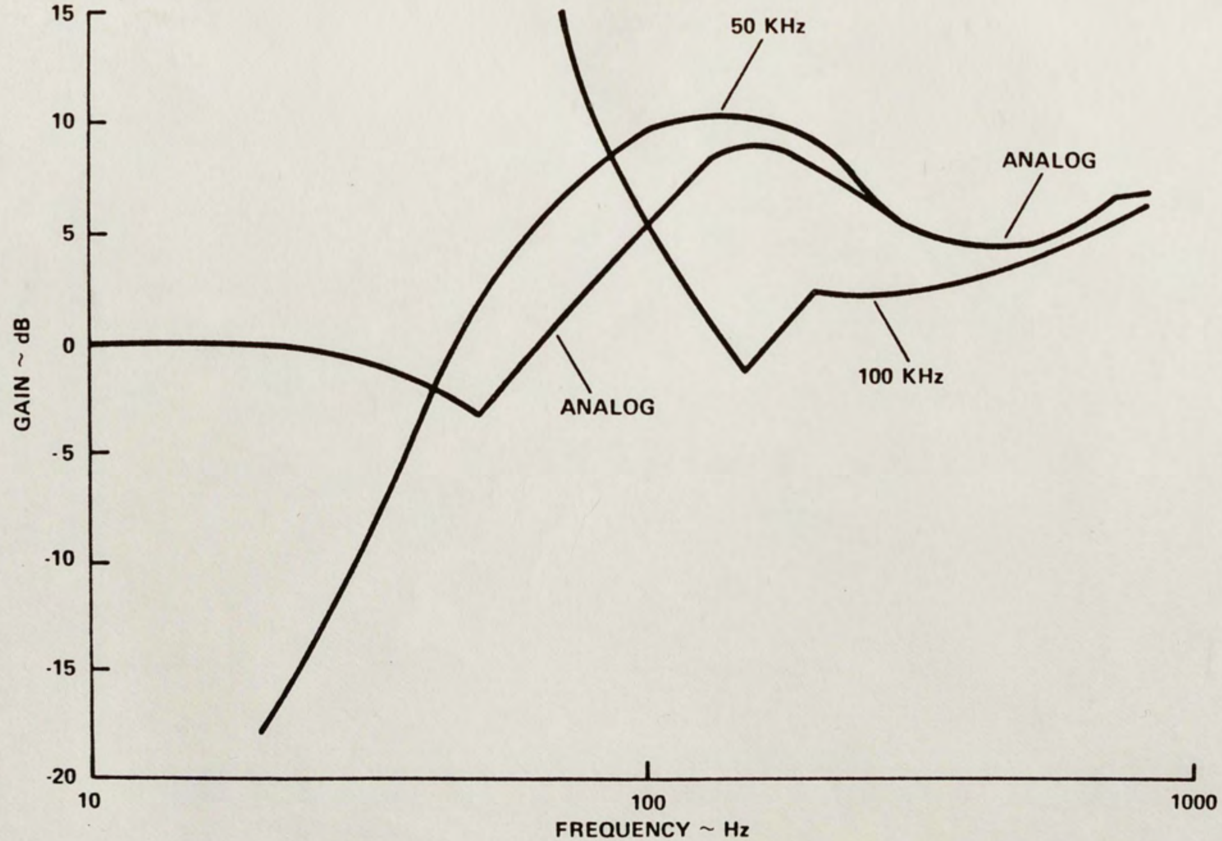

Figure 11. Parametric Digital Equalizer with Truncated Coefficients.

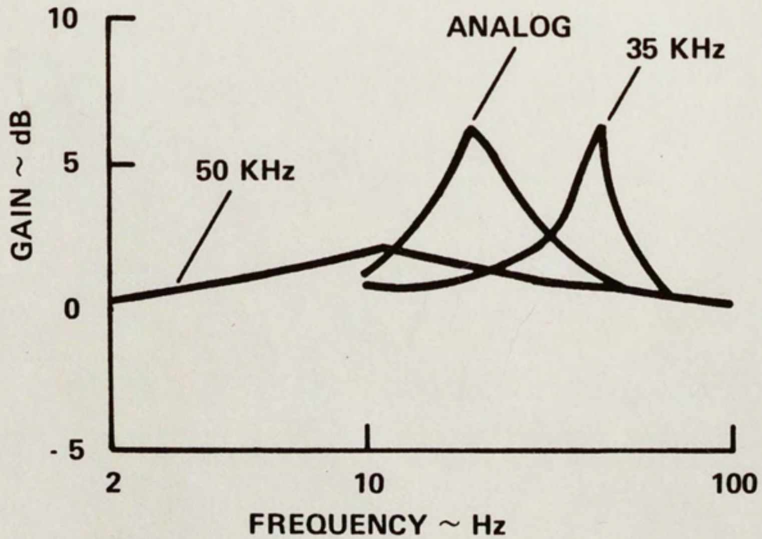

Figure 12. 20 Hz Equalizer Filter with Truncated Coefficients.

This ratio bound assumes that a direct form implementation is used with a coefficient word length of 16 bits. The upper limit of the ratio is based on the 50 Hz filter in the Brubaker and Bullis design (see Table 2). To achieve the 2 dB error, the sample to center frequency ratio must be 500 or less for this filter. A 2 dB error criteria is used instead of the 5 dB high frequency criteria because acoustic responses tend to be more irregular in the low frequency region (see Appendix B).

The lower limit was determined by the fact that the high frequency response error for the 100 Khz digital design is no more than 2 dB which is inaudible. The lower limit is conservative and could be lowered based on the 50 Khz digital design. The 20.2 Khz filter parameters were adjusted to decrease frequency response error due to warping to below 4 dB which is an inaudible response difference at frequencies above 12 Khz.

CHAPTER V

CONCLUSIONS

The primary considerations of the digital equalizer design are the proper choices of coefficient wordlength and sample frequency to maintain an audio bandwidth of 20 Hz to 20 Khz. As the sample frequency is increased, the effect of warping decreases. The problem with increasing the sample rate is that the ability to realize low frequency filters is adversely affected due to finite word length coefficient effects.

A design procedure for minimizing warping and high frequency attenuation from the zero-order hold, anti-aliasing and reconstruction filters is adjustment of the digital biquadratic filter center frequency gain and pole Q. This technique was used with a 50 Khz design to decrease the high frequency error from 8 dB to 1.6 dB.

A sample to center frequency ratio was defined for the biquadratic equalizer filters from frequency responses. Using a 16 bit coefficient quantization level, a range from 5 to 500 is necessary to maintain an inaudible difference between the digital and analog equalizers. At the lower end of the range, warping is the dominant error source. At sample to center frequency ratios of more than 500, coefficient quantization is the dominant error source.

Another design consideration is what type of anti-aliasing and reconstruction filters to use. The design presented here was for an analog input signal; therefore, an anti-aliasing filter should be used. The requirements for the anti-aliasing filter are that it not significantly affect the audible frequencies while eliminating signals above the one-half sample frequency. Audible listening tests should be performed to practically determine the audibility of aliasing in order to specify the anti-aliasing filter. The reconstruction filter requirement also needs to be studied by listening tests. A technique known as oversampling could be used to eliminate the reconstruction filter.

A fixed point simulation of the equalizer with quantization effects must be done to accurately determine the necessary wordlengths and sample rates. Other filter architectures that are less sensitive to coefficient quantization errors need to be considered. The ladder and wave digital filters are much less sensitive than the direct form, but require more memory elements to realize. Using the frequency response analysis, a coefficient wordlength increase of only 4 bits, 16 to 20, is required to use the direct form and cover the frequency range of 20 Hz to 20 Khz using a sample rate of 50 Khz.

It should be clear from what has been presented that a complete "concept-to-hardware" design of a digital equalizer is beyond the scope of a thesis. This claim is verified by the fact that industry has not even achieved this goal. However, it is concluded that the work presented here identifies the more serious problems and indicates some possible approaches to solving them.

APPENDICES

APPENDIX A

MUSICAL INSTRUMENT FREQUENCY RANGES

The frequency ranges for various musical instruments are shown in Figure 13. Figure 13 was reprinted with permission from the DBX, Inc. Automatic Equalizer/Analyzer owner's manual.

From the chart it can be seen that most of the musical energy is well below 5 Khz. There are harmonics which contribute rather significantly to the overall musical sound.

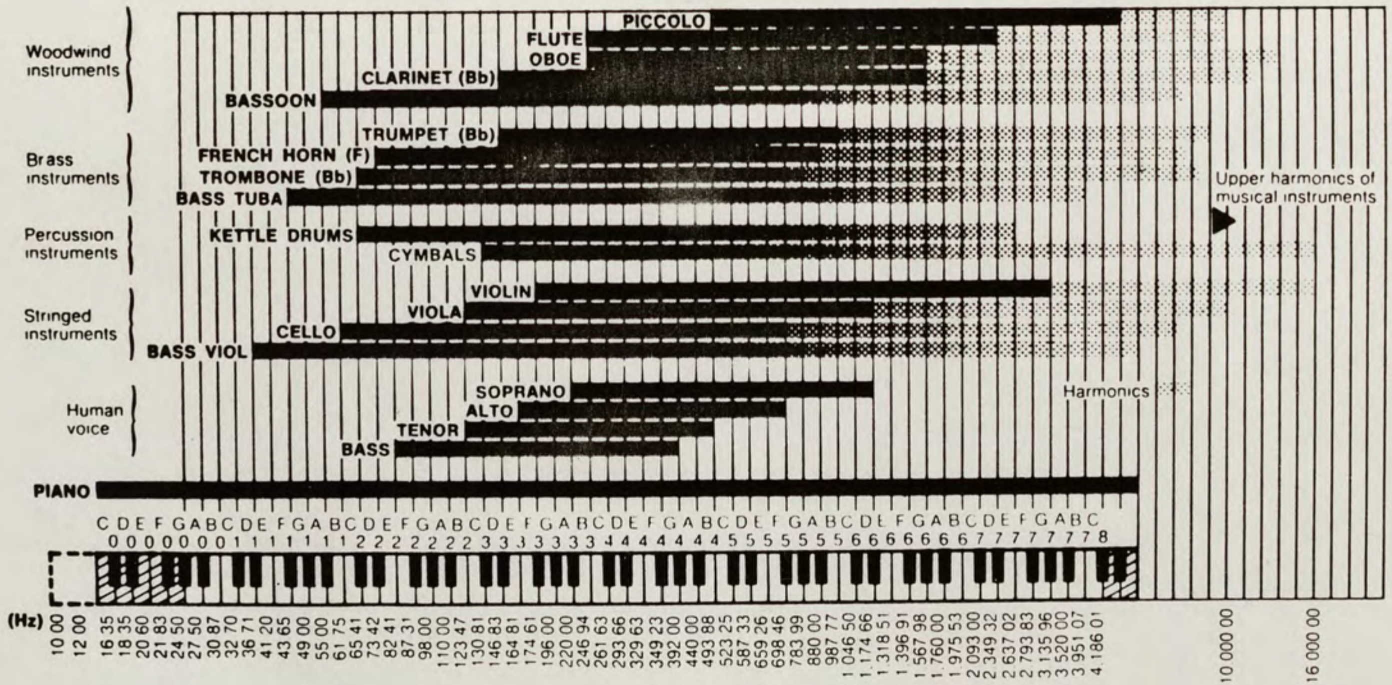

Figure 13. Frequency Ranges of Musical Instruments and the Human Voice.

APPENDIX B

ROOM ACOUSTICS AND FREQUENCY RESPONSE

The following excerpt is taken from the DBX Model 20/20 Computerized Equalizer/Analyzer Owner's Manual.

"Sound waves traveling from a loudspeaker reach a listener by one direct path and many indirect ones. The indirect sound waves must bounce off room surfaces and furnishings one or more times before reaching the listener's ears. These waves are reduced in amplitude each time they are reflected. The amount of reduction depends on the frequencies involved and the characteristics of the surfaces struck. Padded carpeting absorbs mid and high frequencies more than, say, window glass. Panels, doors, walls and other large rigid surfaces can vibrate at low frequencies, sometimes amplifying the sound affecting them. Every material has different absorption characteristics and reacts differently to sound.

"In a room with little material in it to absorb sound, sound waves bounce off the hard surfaces and create multiple echoes (reverberation). This type of room is often referred to as 'live.' Too much reverberation can reduce the clarity of a program. By contrast, a room with much absorptive material is referred to as 'dead.' This type of room will have few echoes and lack brightness.

"The overall effect of this bouncing of sound waves around a room is to produce frequency response irregularities. The shape, dimensions, construction and furnishings all combine to influence the frequency response. Standing waves, which are produced when sound bounces between two parallel walls with little absorption, are the largest contributors to room response anomalies. The wave length of a standing wave is proportional to the dimensions involved. Therefore, larger rooms have lower frequency standing waves. The intensity of a standing wave varies with the amount of absorbent material in the path of the reflecting sounds. The more absorbent the material, the lower the intensity of the standing waves.

"Since standing waves are caused by the interaction of sound with the room boundaries (walls, ceilings, floors, etc.), the perception of the intensity of the standing wave varies with the listener's position in the room. Reinforcement (high intensity) of the lowest frequencies usually occur in the center of the room, while cancellation (low intensity) of these frequencies usually occurs near the walls. At low frequencies, the relatively large response irregularities caused by standing waves tend to be spaced far apart in frequency, therefore, they are quite noticeable. At higher frequencies, however, where standing waves crowd much closer together, the multiple peaks and dips tend to average out, leaving the impression of relatively flat response."

APPENDIX C

ANALYSIS SOFTWARE

The frequency response and filter coefficient generation programs were developed in BASIC. They are modular in that the interface between them is well defined.

EQFRP is the digital equalizer frequency response program. This routine simulates zero-order hold and up to 10 digital filters. The program also generates the responses for the anti-aliasing and reconstruction filters.

The filter coefficients for the analog and digital filters are stored as double precision variables. The frequency response programs currently use single precision for the frequency response generation. The COEFSD program generates the second order analog filter coefficients. The inputs to the program are center frequency gain, center frequency and pole Q. The COEF1SD program generates first order analog filter coefficients.

The COEFZD program generates the second order digital filter coefficients. This routine also applies tangent pre-warping. All of the arithmetic used in this process is double precision. The COEF1ZD program generates the first order filter coefficients. The ZNORM2 program quantizes the digital filter coefficients for first and second order filters. The quantization level is an input variable to ZNORM2.

The frequency response programs are SFREQB and ZFREQA. The SFREQB program is the analog equalizer frequency response routine. It writes the frequency response data to a file for later examination. All of the program control parameters may be entered through a file or interactively. The ZFREQA program generates frequency responses for digital filters.

```
600 '
610 LPRINT "FREQ(HZ)","MAGN(DB)","PHASE(DEG)"
620 FOR K = 1 TO ITER
630 F = F+DELTF
640 W = 2!*PI*F 'compute omega (rad/sec)
650 ' wts=2*pi*f*ts
660 PHASE = 0!:MAG = 1!:WTS = W*TS
670 FOR J=1 TO IFILTZ :'CALCULATE Z FILTER RESPONSES
680 INORD = IORDER(J)
690 GOSUB 880
700 NEXT J
710 FOR J=IFILTZ + 1 TO IFILT :'CALCULATE S FILTER RESPONSES
720 INORD = IORDER(J) ' ASSUME BUMP FILTERS HAVE DIFFERENT ORDERS
730 GOSUB 1130
740 NEXT J
750 ' CALCULATE ZERO ORDER HOLD RESPONSE
760 ZMAG = (2/WTS)*ABS(SIN(WTS/2!))
770 ZPHASE =  - WTS/2!
780 MAG = MAG*ZMAG
790 PHASE = PHASE + ZPHASE
800 '
810 IF MAG>0 THEN MAG=LOG(MAG)/2.30259
820 MAG = MAG*20! 'magnitude in db ????????????????
830 MAG = MAG + GAIN
840 PHASE = PHASE*180!/PI
850 LPRINT F,MAG,PHASE
860 NEXT K
870 GOTO 240
880 'z domain freq response subroutine
890 'numer array is c the denom is d
900 NR=0:NI=0:DI=0:DR=0:
910 FOR IT = 0 TO INORD
920 COSIWTS = COS(IT*WTS)
930 SINIWTS = SIN(IT*WTS)
940 NR = NR + C(IT,J)*COSIWTS
950 NI = NI + C(IT,J)*SINIWTS
960 DR = DR + D(IT,J)*COSIWTS
970 DI = DI + D(IT,J)*SINIWTS
980 NEXT
990 MAGNUM = NR*NR + NI*NI 'magnitude squared
1000 MAGDEN = DR*DR + DI*DI
1010 IF NR=0 THEN NR = 1E-10
1020 PHINUM = ATN(NI/NR)
1030 '
1040 IF DR=0 THEN DR = 1E-10
1050 PHIDEN = ATN(DI/DR)
1060 '
1070 IF NR<0 THEN IF PHINUM<0 THEN PHINUM = PHINUM+PI ELSE
     PHINUM=PHINUM-PI
1080 IF DR<0 THEN IF PHIDEN<0 THEN PHIDEN = PHIDEN+PI ELSE
     PHIDEN=PHIDEN-PI
1090 PHASE = PHINUM - PHIDEN +PHASE
1100 IF PHASE<-PI THEN PHASE = PHASE + 2*PI ELSE IF PHASE>PI THEN
     PHASE=PHASE-2*PI
1110 MAG = SQR(MAGNUM/MAGDEN)*MAG
1120 RETURN
1130 'subroutine to generate s domain freq response inord order
1140 'numer array is C the denom is D
1150 ' w=2*pi*f
1160 NI=0:DI=0:NR=C(0,J):DR=D(0,J)
1170 WNM = 1!
1180 FOR IT=1 TO INORD
1190 WNM = WNM*W
1200 NR = NR + C(IT,J)*WNM*ICOS(IT)
1210 NI = NI + C(IT,J)*WNM*ISIN(IT)
1220 DR = DR + D(IT,J)*WNM*ICOS(IT)
```

```
1230 '
1240 DI = DI + D(IT,J)*WNM*ISIN(IT)
1250 NEXT
1260 MAGNUM = NR*NR + NI*NI  'magnitude squared
1270 MAGDEN = DR*DR + DI*DI
1280 '
1290 IF NR=0 THEN NR=1E-10
1300 PHINUM = ATN(NI/NR)
1310 '
1320 IF DR=0 THEN DR=1E-10
1330 PHIDEN = ATN(DI/DR)
1340 '
1350 IF NR<0 THEN IF PHINUM<0 THEN PHINUM = PHINUM+PI ELSE
     PHINUM=PHINUM-PI
1360 IF DR<0 THEN IF PHIDEN<0 THEN PHIDEN = PHIDEN+PI ELSE
     PHIDEN=PHIDEN-PI
1370 PHASE = PHINUM - PHIDEN +PHASE
1380 IF PHASE<-PI THEN PHASE = PHASE + 2*PI ELSE IF PHASE>PI THEN
     PHASE=PHASE-2*PI
1390 MAG = SQR(MAGNUM/MAGDEN)*MAG
1400 RETURN
```

```
10 'THIS PGM IS COEFSD.BAS IT GENERATES S  COEFFICIENTS FOR
20 ' SECOND ORDER BUMP FILTERS USED IN THE AUDIO EQUALIZER.
30 ' THE BUMP AMPLITUDE IS INPUT IN DB!!!!!!!!!!!!!!!!!!!!!
40 ' THE BUMP FREQUENCY IS INPUT IN HZ
50 ' THE PGM WAS CREATED 1/27/85 BY TNT
60 ' THE GAIN FOR THE 2ND ORDER BUMP FILTER IS 0 DB
70 ' THE OUTPUT FILE FORMAT IS COMPATIBLE WITH COEFZD 1/27/85
80 DEFDBL A-H,M-Z
90 DEFINT I-L:PI=3.141592654#:DIM A(2),B(2)
100 GAIN = 0 ' INVERSE DC FILTER GAIN FOR 2ND ORDER BUMP FILTER
110 '
120 ' FO IS THE BUMP FREQ - MDB IS BUMP HEIGHT IN DB - QP IS POLE Q
130 INPUT "enter FO,A(DB),QP for the 2nd order bump filter";FO,MDB,QP
140 M = MDB/20#
150 M=10#^M
160 'COMPUTE S COEFFICIENTS
170 A(2) = 1# ' s numerator
180 B(2) = 1# ' s denominator
190 A(1) = M*2#*PI*FO/QP
200 B(1) = 2#*PI*FO/QP
210 A(0) = 4#*PI*PI*FO^2
220 B(0) = A(0)
230 ' WRITE S COEFFICIENTS
240 PRINT "A=";M,"QP=";QP,"FO=";FO
250 PRINT "numer";"          denom s coefficients"
260 FOR I = 0 TO 2:PRINT A(I),B(I):NEXT
270 INPUT "enter 1 to save the coefficients on disk";IFLAG
280 IF IFLAG<>1 THEN STOP
290 LINE INPUT "enter the output filename"; FLNAME$
300 LINE INPUT "enter title info for the file";T$
310 OPEN "O",1,FLNAME$
320 PRINT #1,FLNAME$
330 WRITE #1, 2
340 FOR I = 0 TO 2:WRITE #1,A(I),B(I):NEXT
350 PRINT #1,"BUMP FREQ(HZ) ";FO;" BUMP AMPL(DB) ";MDB;" POLE Q ";QP
360 PRINT #1,T$
370 WRITE #1,FS,GAIN,QP,FO
380 CLOSE #1
```

```
10 'THIS PGM IS COEFZD.BAS IT GENERATES Z COEFFICIENTS FROM S COEFS
20 'FOR SECOND ORDER BUMP FILTERS USED IN THE AUDIO EQUALIZER.
30 'THE COEFFICIENTS ARE NOT NORMALIZED BY F(2)!!!!!!!!!!!!!
40 ' THE PGM WAS CREATED 1/27/85 BY TNT
50 ' THE GAIN FOR THE 2ND ORDER BUMP FILTER IS 0 DB
60 DEFDBL A-H,M-Z
70 DEFINT I-L:PI=3.141592654#:DIM A(2),B(2),E(2),F(2)
80 INPUT "enter sample frequency(hz)";FS
90 INPUT "ENTER INPUT FILENAME ";FLNAM1$
100 ' READ S FILE!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
110 OPEN "I",1,FLNAM1$
120 LINE INPUT #1,FLNAM$:INPUT #1,IORDER
130 FOR I = 0 TO IORDER:INPUT #1,A(I),B(I):NEXT I
140 LINE INPUT #1,FINFO$' PERTINENT FILTER INFO
150 LINE INPUT #1,TITLE$:' TITLE INFO
160 INPUT #1,FSDUM,GAIN,QP,FO
170 CLOSE #1
180 '
190 FOW = FS*TAN(PI*FO/FS)/PI 'warped frequency in s domain(hz)
200 PRINT "the warped bump freq(hz) is ";FOW
210 RATIO = FOW/FO ' RATIO OF WARPED FREQ TO UNWARPED
220 RATIO2 = RATIO*RATIO
230 A(1) = A(1) * RATIO
240 B(1) = B(1) * RATIO
250 A(0) = A(0) * RATIO2
260 B(0) = B(0) * RATIO2
270 'THE WARPED FREQ COEFFICIENTS ARE USED FOR THE Z FUNCTIONS
280 'COMPUTE Z COEFFICIENTS
290 TO2 = 1#/(2#*FS): TO22 = TO2*TO2 'T/2 and T/2 squared
300 E(2) = 1# + TO2*A(1) + TO22*A(0) ' z numerator
310 F(2) = 1# + TO2*B(1) + TO22*B(0) ' z denominator
320 E(1) = 2#*(TO22*A(0) - 1#)
330 F(1) = 2#*(TO22*B(0) - 1#)
340 E(0) = 1# - TO2*A(1) + TO22*A(0)
350 F(0) = 1# - TO2*B(1) + TO22*B(0)
360 'DO NOT NORMALIZE THE COEFICIENTS BY F(2)
370 ' FOR I = 0 TO 2:E(I) = E(I)/F(2):F(I) = F(I)/F(2):NEXT
380 ' WRITE Z COEFFICIENTS
390 PRINT "FO=";FO,"FO warped=";FOW
400 T2$ = "SAMPLE FREQ "
410 PRINT "numer";"        denom z coefficients"
420 FOR I = 0 TO IORDER: PRINT E(I),F(I):NEXT
430 INPUT "enter 1 to save the coefficients on disk";IFLAG
440 IF IFLAG<>1 THEN 540
450 LINE INPUT "enter the output filename"; FLNAME$
460 OPEN "O",1,FLNAME$
470 PRINT #1,FLNAME$
480 WRITE #1, IORDER
490 FOR I = 0 TO IORDER:WRITE #1,E(I),F(I):NEXT
500 PRINT #1,FINFO$ + T2$;FS
510 PRINT #1,TITLE$
520 WRITE #1,FS,GAIN,QP,FOW
530 CLOSE #1
540 STOP
```

```
10 ' THE PGM NAME IS ZNORM2.BAS CREATED 3/23/85
20 ' THIS PGM NORMALIZES Z FILTERS FOR ANALYSIS OF COEFFICIENT
30 ' QUANTIZATION ERRORS
40 ' THE NORMALIZATION IS PERFORMED BY DIVIDING THE COEFFICIENTS BY
50 ' THE COEFFICIENT OF THE HIGHEST POWER OF Z IN THE DENOMINATOR
60 ' MODIFIED 3/23/85 TO USE DOUBLE PRECISION ???????????????
70 ' AND TO WRITE THE NORMALIZED COEFS TO A FILE!!!!!!!!!!!!!
80 '
90 ' MODIFIED 3/23/85 TO STORE NORMALIZED COEF'S IN SINGLE PRECISION
100 ' VARIABLE TO MORE ACCURATELY REFLECT 16 BIT EFFECT
110 ' MODIFIED 3/28/85 TO ROUND COEFS TO FINITE WORD LENGTH
120 ' LEVEL
130 DEFINT I-L:DEFDBL A-H,M-Z
140 LINE INPUT "ENTER INPUT FILENAME ";FLNAME$
150 INPUT "ENTER NUMBER OF FRACTIONAL BITS "; IBIT
160 FRAC = 2^ -IBIT
170 ' read coefficient data from the disk
180 OPEN "I",1,FLNAME$
190 LINE INPUT #1,FLNAM$:INPUT #1,IORDER
200 FOR I = 0 TO IORDER:INPUT #1,C(I),D(I):NEXT I
210 LINE INPUT #1, FINFO$:' PERTINENT FILTER INFO
220 LINE INPUT #1,TITLE$:' TITLE INFO
230 INPUT #1,FS,GAIN,QP,FOW
240 CLOSE #1
250 ' STORE NORMALIZED COEFFICIENTS IN SINGLE PRECISION VARIABLE
260 DX3 = D(IORDER)
270 FOR I = 0 TO IORDER
280 C!(I) = C(I) / DX3
290 D!(I) = D(I) / DX3
300 NEXT I
310 'ROUND COEFFICIENTS
320 FOR I = 0 TO IORDER
330 'NUMER COEFS
340 TEMP = 2 ^ IBIT * (C!(I))
350 BIT = TEMP - INT(TEMP)
360 IF BIT > .5 THEN C!(I) = INT(TEMP) + 1:ELSE C!(I) = INT(TEMP)
370 C!(I) = FRAC * C!(I)
380 ' DENOM COEFS
390 TEMP = 2 ^ IBIT * (D!(I))
400 BIT = TEMP - INT(TEMP)
410 IF BIT > .5 THEN D!(I) = INT(TEMP) + 1:ELSE D!(I) = INT(TEMP)
420 D!(I) = FRAC * D!(I)
430 NEXT I
440 PRINT " FILENAME  " + FLNAM$ + "  WITH ROUNDED Z COEFFICIENTS "
450 PRINT " USING "; IBIT; " FRACTIONAL BITS "
460 FOR I = 0 TO IORDER:PRINT C!(I),D!(I):NEXT I
470 PRINT FINFO$:' PERTINENT FILTER INFO
480 PRINT TITLE$:' TITLE INFO
490 INPUT "enter 1 to save the coefficients on disk";IFLAG
500 IF IFLAG<>1 THEN 600
510 LINE INPUT "enter the output filename"; FLNAME$
520 OPEN "O",1,FLNAME$
530 PRINT #1,FLNAME$
540 WRITE #1, IORDER
550 FOR I = 0 TO IORDER:WRITE #1,C!(I),D!(I):NEXT
560 PRINT #1,FINFO$
570 PRINT #1,TITLE$ + " 16 BIT "
580 WRITE #1,FS,GAIN,QP,FOW
590 CLOSE #1
600 STOP
```

```
10 ' THE PGM NAME IS SFREQB.BAS CREATED 6/1/85 BY TNT
20 '
30 ' THE INDIVIDUAL FILTER GAINS ARE IGNORED
40 ' THIS PGM GENERATES FREQ. RESPONSES OF S FILTERS AND STORES THE
50 ' RESPONSES IN A FILE DENOTED BY UNIT #1
60 '
70 DEFINT I-L:PI=3.14159:DIM A(10,10),B(10,10),IORDER(10)
80 DIM ICOS(6),ISIN(6)
90 'data for the cosine  array follows
100 DATA 0,-1,0,1,0,-1
110 'data for sine array follows
120 DATA 1,0,-1,0,1,0
130 RESTORE 100
140 FOR I=1 TO 6:READ ICOS(I):NEXT:
150 FOR I=1 TO 6:READ ISIN(I):NEXT:
160 '
170 LINE INPUT "ENTER NAME OF THE RUN PARAMETER FILE";FLINPU$
180 '
190 'OPEN INPUT PARAMETER FILE #2
200 OPEN "I",2,FLINPU$: 'INPUT FILE WITH RUN PARAMETERS
210 LINE INPUT #2, FLNAM1$ 'OUTPUT FILENAME
220 INPUT #2, GAINI
230 GAINF1 = 0
240 INPUT #2,IFILT
250 ' read coefficient data from the disk
260 FOR J=1 TO IFILT :LINE INPUT #2,FLNAME$(J):NEXT J
270 FOR J= 1 TO IFILT : OPEN "I",1,FLNAME$(J)
280 LINE INPUT #1,FLNAM$:INPUT #1,IORDER(J)
290 FOR I = 0 TO IORDER(J):INPUT #1,A(I,J),B(I,J):NEXT I
300 LINE INPUT #1,FINFO$(J)' PERTINENT FILTER INFO
310 LINE INPUT #1,TITLE$(J):' TITLE INFO
320 INPUT #1,FS,GAINF,DUM1,DUM2
330 CLOSE #1:NEXT J
340 GAIN = GAINI ' GAIN IS USED IN THE PGM FOR THE FINAL GAIN COMP
350 '
360 'OPEN OUTPUT FILE FOR FREQUENCY RESPONSE
370 '
380 OPEN "O",1,FLNAM1$
390 WRITE #1, IFILT
400 FOR J= 1 TO IFILT
410 PRINT #1,FINFO$(J):   ' PERTINENT FILTER INFO
420 PRINT #1,TITLE$(J):  ' TITLE INFO
430 NEXT J
440 ' READ F1,F2,DELTF FROM FILE #2  COMPUTE ITER
450 INPUT #2, F1,F2,DELTF
460 IF F1=-99 THEN WRITE #1,F1,F2,DELTF: CLOSE :STOP
470 ITER = (F2-F1)/DELTF + 1
480 F = F1 - DELTF
490 FOR K = 1 TO ITER
500 F = F+DELTF
510 W = 2*PI*F 'compute omega (rad/sec)
520 PHASE=0!:MAG=1!' initialization for the magnitude and phase
530 FOR J= 1 TO IFILT
540 INORD = IORDER(J) ' ASSUME BUMP FILTERS HAVE DIFFERENT  ORDERS
550 GOSUB 660
560 NEXT J
570 IF MAG>0 THEN MAG=LOG(MAG)/2.30259
580 MAG = MAG*20 'magnitude in db ????????????????
590 MAG = MAG + GAIN
600 PHASE = PHASE*180!/PI
```

```
610 '
620 '
630 WRITE #1, F,MAG,PHASE
640 NEXT K
650 GOTO 450
660 'subroutine to generate s domain freq response inord order
670 'numer array is a the denom is b
680 ' w=2*pi*f
690 NI=0:DI=0:NR=A(0,J):DR=B(0,J)
700 WNM = 1!
710 FOR IT=1 TO INORD
720 WNM = WNM*W
730 NR = NR + A(IT,J)*WNM*ICOS(IT)
740 NI = NI + A(IT,J)*WNM*ISIN(IT)
750 DR = DR + B(IT,J)*WNM*ICOS(IT)
760 DI = DI + B(IT,J)*WNM*ISIN(IT)
770 NEXT
780 MAGNUM = NR*NR + NI*NI  'magnitude squared
790 MAGDEN = DR*DR + DI*DI
800 '
810 IF NR=0 THEN NR=1E-10
820 PHINUM = ATN(NI/NR)
830 '
840 IF DR=0 THEN DR=1E-10
850 PHIDEN = ATN(DI/DR)
860 '
870 IF NR<0 THEN IF PHINUM<0 THEN PHINUM = PHINUM+PI
    ELSE PHINUM=PHINUM-PI
880 IF DR<0 THEN IF PHIDEN<0 THEN PHIDEN = PHIDEN+PI
    ELSE PHIDEN=PHIDEN-PI
890 PHASE = PHINUM - PHIDEN +PHASE
900 IF PHASE<-PI THEN PHASE = PHASE + 2*PI ELSE IF PHASE>PI THEN
    PHASE=PHASE-2*PI
910 MAG = SQR(MAGNUM/MAGDEN)*MAG
920 RETURN
```

```
10 ' THE PGM NAME IS ZFREQA.BAS CREATED 1/85
20 'THIS IS THE DIGITAL FILTER FREQUENCY RESPONSE PGM
30 ' MODIFIED 1/15/85 TO ACCEPT NEW COEFSZ OUTPUT
40 ' ALL REAL SINGLE PRECISION VARIABLES ARE USED IN THIS
50 ' VERSION OF ZFREQ 1/16/85
60 ' MODIFIED 1/26/85 TO ACCEPT NEW COEFSZD FILE FORMAT
70 ' THE NEW FORMAT HAS SAMPLE FREQUENCY AND INVERSE FILTER DC GAIN
80 DEFINT I-L:PI=3.14159:DIM C(10,10),D(10,10),FLNAME$(10),TITLE$(10)
90 DIM FINFO$(10),IORDER(10)
100 GOTO 120
110 STOP
120 INPUT "enter 1 to read data from disk";IFLAG
130 INPUT "enter start freq,end freq,freq step";F1,F2,DELTF
140 INPUT "ENTER GAIN FACTOR (DB) "; GAINI
150 ITER = (F2-F1)/DELTF + 2
160 F=F1 - DELTF
170 'inord is the order of the transfer function USED IN THE PGM
180 IF IFLAG<>1 GOTO 340
190 GAINF1 = 0 ' INVERSE FILTER GAIN (DC) VARIABLE
200 INPUT "enter the number of filter sections desired";IFILT
210 ' read coefficient data from the disk
220 FOR J=1 TO IFILT :LINE INPUT "enter filename(s)";FLNAME$(J):NEXT J
230 FOR J= 1 TO IFILT : OPEN "I",1,FLNAME$(J)
240 LINE INPUT #1,FLNAM$:INPUT #1,IORDER(J)
250 FOR I = 0 TO IORDER(J):INPUT #1,C(I,J),D(I,J):NEXT I
260 LINE INPUT #1, FINFO$(J):' PERTINENT FILTER INFO
270 LINE INPUT #1,TITLE$(J):' TITLE INFO
280 INPUT #1,FS,GAINF,DUM1,DUM2
290 TS = 1!/FS' CHECK TO SEE THAT ALL FILTERS ARE FOR THE SAME
    SAMPLE FREQUENCY
300 IF J=1 THEN FS1 = FS
310 IF J>1 THEN IF FS<>FS1 THEN PRINT "THE INPUT FILES WERE
    GENERATED WITH DIFFERENT SAMPLE FREQUENCIES ??????":STOP
320 GAINF1 = GAINF1 + GAINF ' INVERSE FILTER DC GAIN FOR ALL FILTERS
330 CLOSE #1:NEXT J
340 GAIN = GAINF1 + GAINI ' GAIN IS THE FINAL GAIN COMPUTATION
350 FOR I=1 TO IFILT:LPRINT FINFO$(I),FLNAME$(I)
360 LPRINT TITLE$(I):NEXT I
370 LPRINT "DATE RAN ";DATE$;" THE TIME IS ";TIME$
380 LPRINT "FREQ(HZ)","MAGN(DB)","PHASE(DEG)"
390 '
400 FOR K = 1 TO ITER
410 F = F+DELTF
420 W = 2!*PI*F 'compute omega (rad/sec)
430 ' wts=2*pi*f*ts
440 PHASE = 0!:MAG = 1!:WTS = W*TS
450 FOR J=1 TO IFILT
460 INORD = IORDER(J) ' ASSUME ALL BUMP FILTERS HAVE DIFFERENT ORDERS
470 GOSUB 560
480 NEXT J
490 IF MAG>0 THEN MAG=LOG(MAG)/2.30259
500 MAG = MAG*20! 'magnitude in db ???????????????
510 MAG = MAG + GAIN
520 PHASE = PHASE*180!/PI
530 LPRINT F,MAG,PHASE
540 NEXT K
550 GOTO 110
560 'z domain freq response subroutine
570 'numer array is c the denom is d
580 NR=0:NI=0:DI=0:DR=0:
590 '
```

```
590
600 '
610 FOR IT = 0 TO INORD
620 COSIWTS = COS(IT*WTS)
630 SINIWTS = SIN(IT*WTS)
640 NR = NR + C(IT,J)*COSIWTS
650 NI = NI + C(IT,J)*SINIWTS
660 DR = DR + D(IT,J)*COSIWTS
670 DI = DI + D(IT,J)*SINIWTS
680 NEXT
690 MAGNUM = NR*NR + NI*NI 'magnitude squared
700 MAGDEN = DR*DR + DI*DI
710 IF NR=0 THEN NR = 1E-10
720 PHINUM = ATN(NI/NR)
730 '
740 IF DR=0 THEN DR = 1E-10
750 PHIDEN = ATN(DI/DR)
760 '
770 IF NR<0 THEN IF PHINUM<0 THEN PHINUM = PHINUM+PI
    ELSE PHINUM=PHINUM-PI
780 IF DR<0 THEN IF PHIDEN<0 THEN PHIDEN = PHIDEN+PI
    ELSE PHIDEN=PHIDEN-PI
790 PHASE = PHINUM - PHIDEN +PHASE
800 IF PHASE<-PI THEN PHASE = PHASE + 2*PI ELSE
    IF PHASE>PI THEN PHASE=PHASE-2*PI
810 MAG = SQR(MAGNUM/MAGDEN)*MAG
820 RETURN
```

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